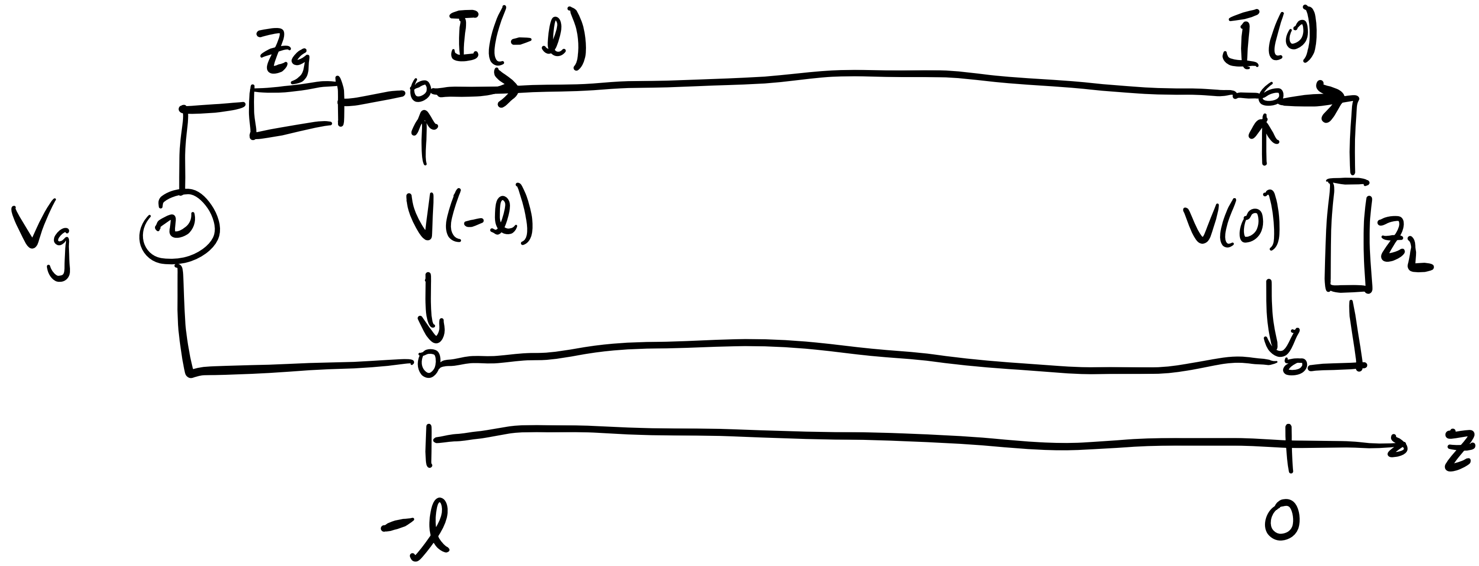
PHYS 331 - Nov. 20, 2023

Useful results from earlier in the term:

1. Transmission line results for amplitude of harmonic voltage & current



Frequency Domain

$$V(z,\omega) = V_+\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right] \equiv \hat{v}(z,\omega) \quad \text{\#}$$

$$I(z,\omega) = \frac{V_+}{Z_0}\left[e^{-j\beta z} - \Gamma e^{j\beta z}\right] \equiv \hat{i}(z,\omega) \quad *$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \beta = \omega\sqrt{L_\ell C_\ell} = \frac{\omega}{s}$$

prop. speed.

② Inverse Fourier Transforms

(a) $$f(t) = F^{-1}\left[\hat{f}(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega) e^{j\omega t}\, d\omega$$

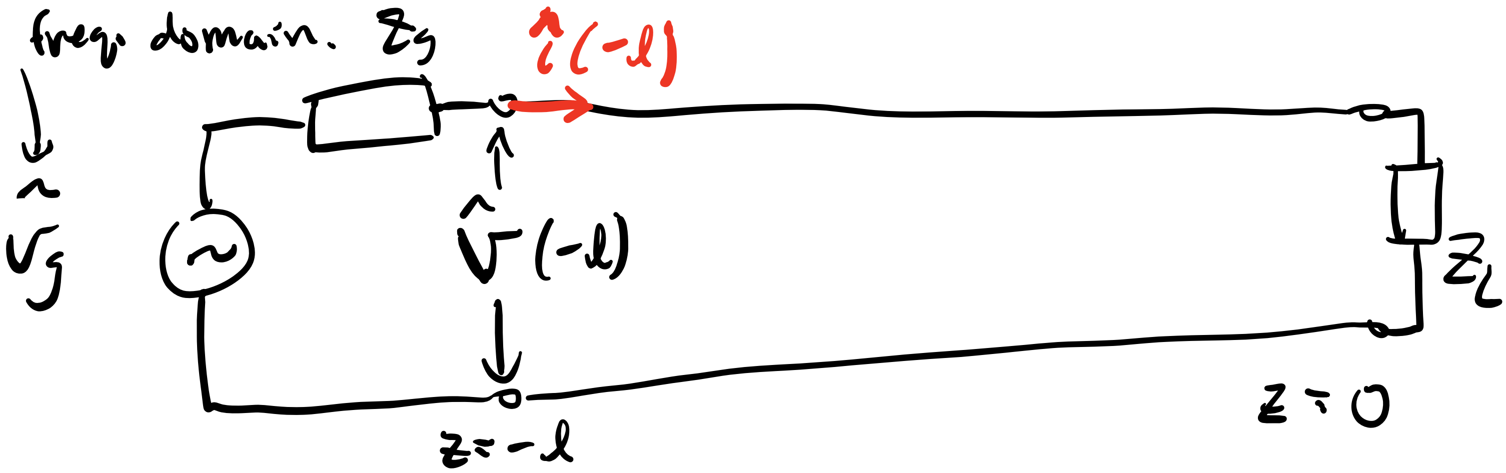
(b) Delta fcn: $$\delta(t-t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega(t-t_0)}\, d\omega$$

(c) Convolution Theorem

$$F^{-1}\left[\hat{x}_1(\omega)\, \hat{x}_2(\omega)\right] = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

Goal is to calc. transient response (time domain) of a transmission line.
Strategy: Employ the freq. domain results that we already know. Then take inverse Fourier transform to convert to time domain.



$\beta = \frac{\omega}{s}$ ← speed

$$\hat{v}_g - \hat{i}(-l) Z_g = \hat{v}(-l)$$

use (*) for $\hat{i}$ w/ $z=-l$

$$\hat{v}_g - \frac{V_+}{Z_0}\left[e^{j\omega l/s} - \Gamma e^{-j\omega l/s}\right] Z_g = \hat{v}(-l)$$

$$= V_+\left[e^{j\omega l/s} + \Gamma e^{-j\omega l/s}\right]$$

using (#)

~~So~~ Solve for V_+ (unknown)

$$V_+ = \frac{\hat{v}_g}{\left(e^{j\omega l/s} + \cancel{\Gamma e^{-j\omega l/s}}\right) + \cancel{\frac{Z_g}{Z_0}}^{\,1}\left(e^{j\omega l/s} - \cancel{\Gamma e^{-j\omega l/s}}\right)}$$

Assume that $Z_g = Z_0$ s.t. $\frac{Z_g}{Z_0} = 1$.

In this case,

$$\boxed{V_+ = \frac{\hat{V}_g}{2} e^{-j\omega l/s}}$$

For this special case, the volt. at the trans. line input in the freq. domain is $\hat{V}_{in} = \hat{V}(-l)$

$$\hat{V}_{in} = V_+ \left[e^{j\omega l/s} + \Gamma e^{-j\omega l/s} \right] \quad \#$$

$$= \frac{\hat{V}_g}{2} \left[1 + \Gamma e^{-2j\omega l/s} \right] \quad !$$

Suppose we meas. the volt. at trans. line input in the time domain using an osc. In that case, we expect to meas. the inverse Fourier trans. form of $\hat{V}_{in}$ to get $V_{in}(t)$.

$$V_{in}(t) = F^{-1}\left[\hat{V}_{in}(\omega) \right] = F^{-1}\left[\frac{\hat{V}_g}{2} \left(1 + \Gamma e^{-2j\omega l/s} \right) \right]$$

Let's assume that Γ is indep. of freq.

$$v_{in}(t) = \frac{1}{2} \underbrace{F^{-1}\left[\hat{V}_g\right]}_{v_g(t)} + \frac{\Gamma}{2} F^{-1}\left[\underbrace{\hat{V}_g}_{\hat{X}_1(\omega)} \underbrace{e^{-2j\omega l/s}}_{\hat{X}_2(\omega)}\right]$$

apply convolution theorem.

$$F^{-1}\left[\hat{V}_g\right] * F^{-1}\left[e^{-2j\omega l/s}\right]$$

$$= v_g(t) * F^{-1}\left[e^{-2j\omega l/s}\right]$$

Note:

$$F^{-1}\left[e^{-2j\omega l/s}\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-2j\omega l/s} e^{j\omega t}\, d\omega$$

$$\underbrace{= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega(t - \underbrace{2l/s}_{t_0})}\, d\omega}$$

$$= \delta(t - t_0) = \delta(t - 2l/s)$$

Note:

$$V_g(t) * F^{-1}\left[e^{-2j\omega l/s}\right]$$

$$= V_g(t) * \delta(t - 2l/s)$$

$$= \int_{-\infty}^{\infty} V_g(\tau)\, \delta\left[(t - 2l/s) - \tau\right] d\tau$$

Recall $\int f(t)\,\delta(t - t_0)\,dt = f(t_0)$

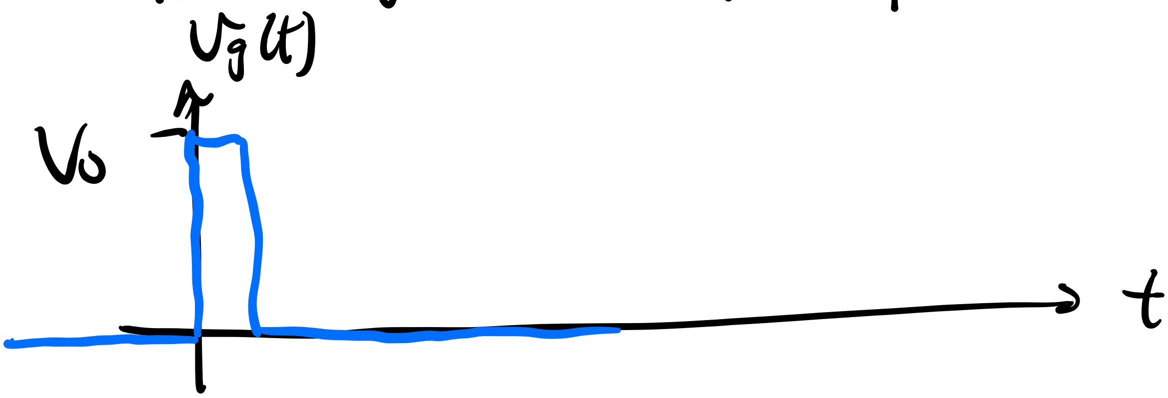
$$= V_g(t - 2l/s)$$

represents a delayed pulse arriving at input after reflecting from end of trans. line.

Bringing all of these results together:

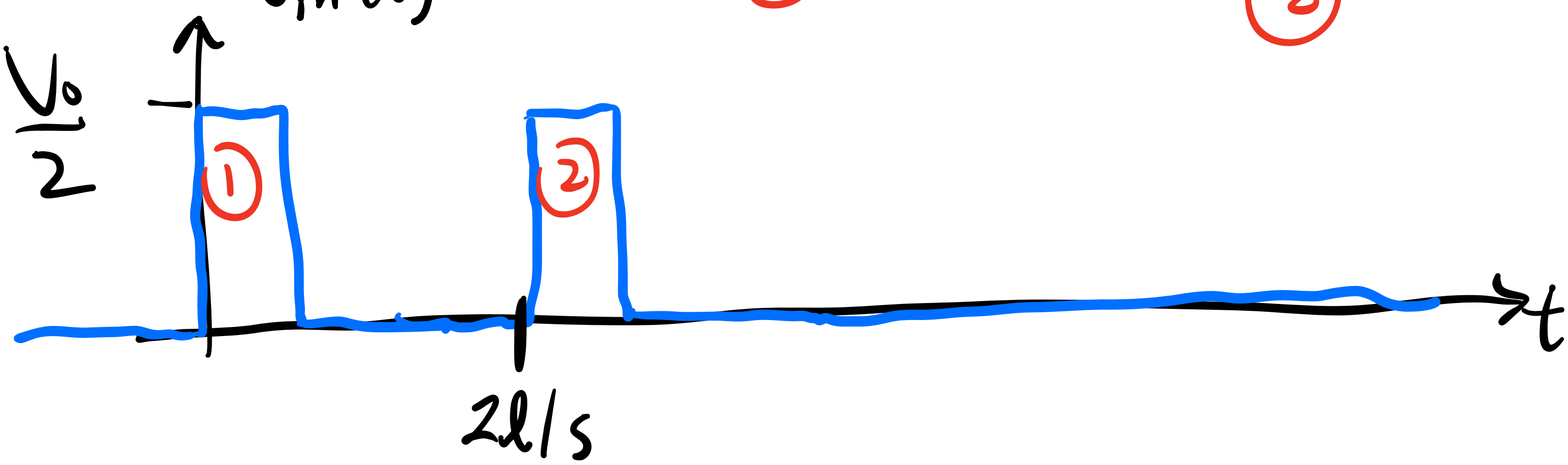
$$\boxed{v_{in}(t) = \frac{v_g(t)}{2} + \frac{\Gamma}{2} v_g(t - 2l/s)}$$

Suppose $v_g(t)$ is a square pulse



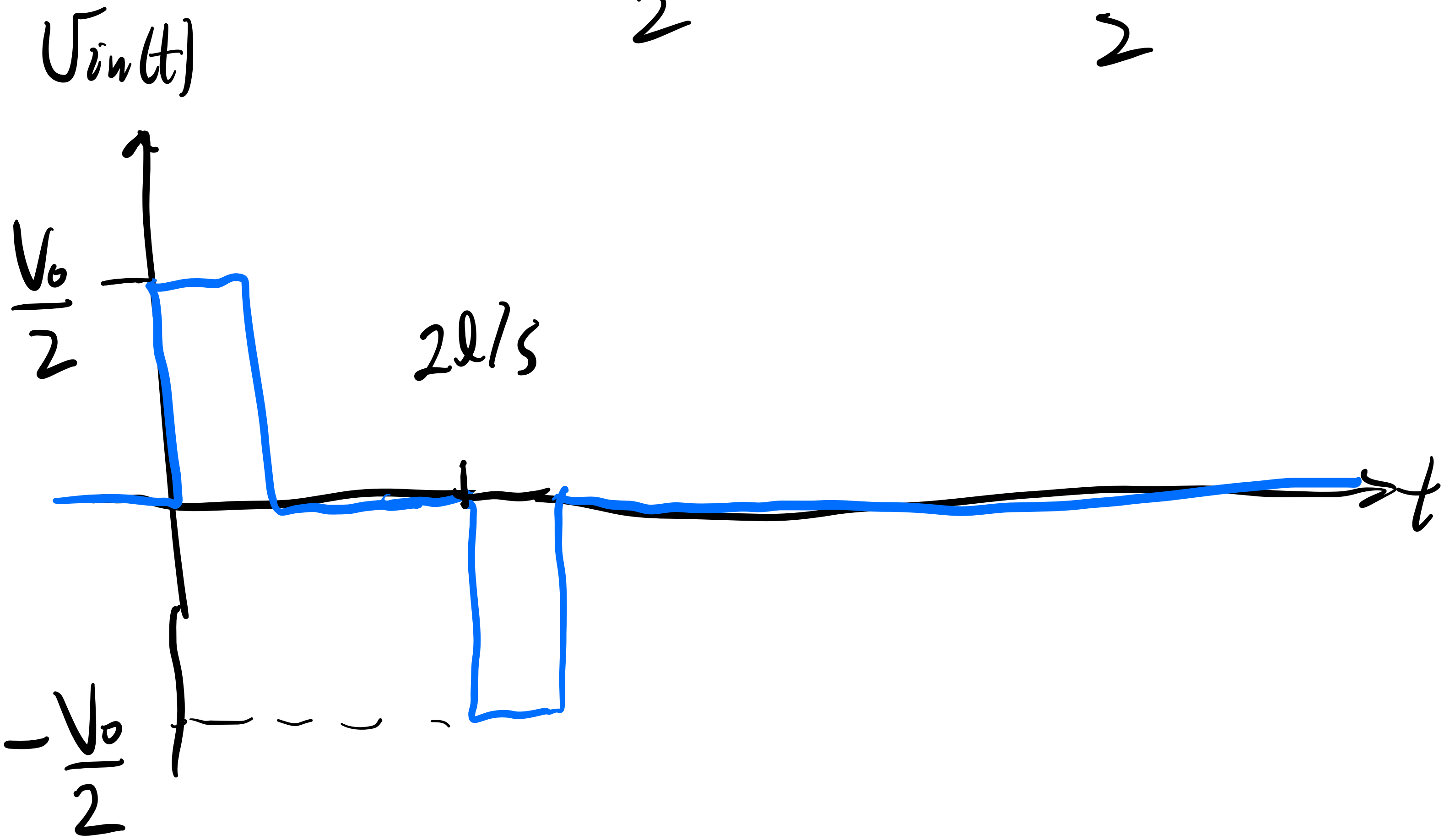
Case 1. $\Gamma = 1$ ($Z_L \to \infty$, open circuit)

$$v_{in}(t) = \underbrace{\frac{v_g(t)}{2}}_{1} + \underbrace{\frac{v_g(t - 2l/s)}{2}}_{2}$$



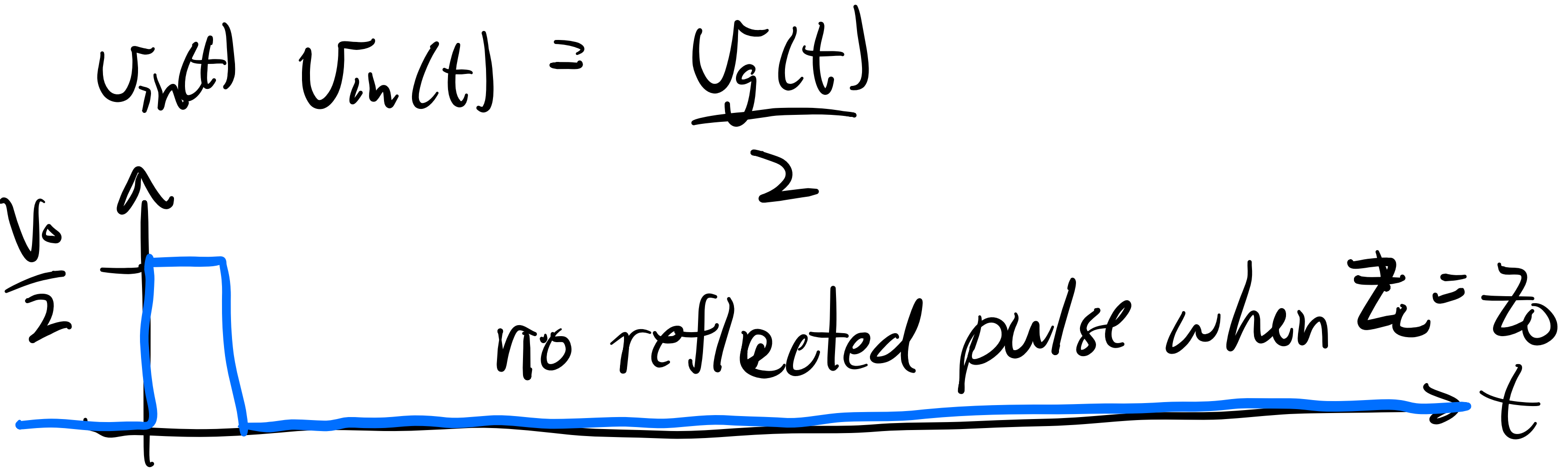
Case 2. $\Gamma = -1$ ($Z_L = 0$, short circuit)

$$V_{in}(t) = \frac{V_g(t)}{2} - \frac{V_g(t - 2l/s)}{2}$$



Case 3. $\Gamma = 0$ ($Z_L = Z_0$)

$$V_{in}(t) = \frac{V_g(t)}{2}$$



no reflected pulse when $Z_L = Z_0$.